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Locally nonlinear planar waveguide with girotropy

A. ZAGÓRSKI, P. SŁOWIKOWSKI

Institute of Physics, Warsaw University of Technology, ul. Koszykowa 75, 00–662 Warszawa, Poland.

In the paper, some results concerning light propagation in a planar waveguide with a local nonlinearity are presented. External magnetic field is assumed as a source of girotropy. A strong laser beam perpendicular to the film couples with the transmitted wave via the nonlinear medium. Both fields influence the outgoing wave leading to some essential changes in its polarization. The parameters characterizing this polarization have been calculated and discussed for several interesting reslizations.

1. Physical model and basic equations

In many optoelectronical devices one uses planar waveguides containing regions with local nonlinear and/or girotropic sections [1], [2]. A combination of these two factors essentially influences the polarization of the outgoing wave which itself is formed by nonlinear interactions. In this paper, we report on some results concerning light propagation in a waveguide with the third-order nonlinearity subjected additionally to an external magnetic field B parallel to the film considered. This field is, in fact, the source of the girotropy.

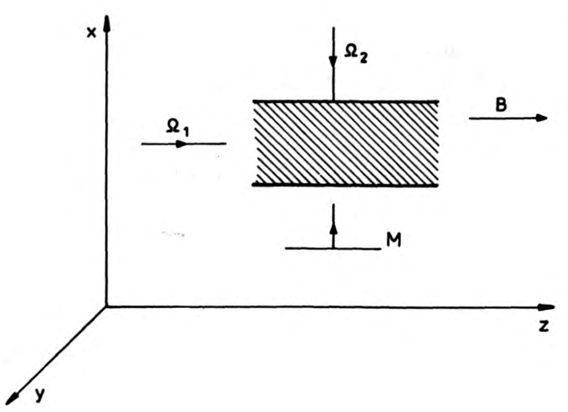

Fig. 1. Geometry of the system

The geometry of our system is presented in Figure 1. A laser beam of the frequency Ω_2 is reflected by a mirror and these two waves evoke a nonlinear behaviour of the film medium. This leads to a nonlinear term in the polarization vector, usually denoted by $\boldsymbol{P}^{\mathrm{NL}}$. We assume the simplest form of this term, namely

$$\boldsymbol{P}^{\mathrm{NL}} = \chi E^2 \boldsymbol{E} \tag{1}$$

with constant susceptibility χ. This expression adds to the linear polarization $\boldsymbol{P}^{\mathrm{L}}$, given by

$$\boldsymbol{P}^{\mathrm{L}} = (\varepsilon-1)\boldsymbol{E} = \begin{bmatrix} \eta, & -i\alpha B, & 0 \\ i\alpha B, & \eta, & 0 \\ 0, & 0, & \eta+\gamma B^2 \end{bmatrix} \boldsymbol{E} \tag{2}$$

where η, α (the girotropic constant) and γ are treated as material constants, independent of external fields. Their values will be discussed at the end of the paper.

The equation for $\boldsymbol{E}$

$$\mathrm{rot\ rot}\ \boldsymbol{E} = -\mu_0 \frac{\partial^2}{\partial t^2} \boldsymbol{D} \tag{3}$$

is normally solved by means of the substitution

$$\boldsymbol{E} = \sum_j \boldsymbol{E}_j e^{-i\omega_j t} \tag{4}$$

where j denotes possible modes of the undisturbed waveguide. The amplitudes $\boldsymbol{E}_j$ fulfil the equation

$$\mathrm{rot\ rot}\ \boldsymbol{E}_j - \frac{\omega_j^2}{c^2}\varepsilon \boldsymbol{E}_j = \mu_0 \omega_j^2 \chi \sum_{lmn}{}' (\boldsymbol{E}_l \boldsymbol{E}_m) \boldsymbol{E}_n. \tag{5}$$

Here the last sum runs over all components satisfying the condition

$$\omega_l + \omega_m + \omega_n = \omega_j \tag{6}$$

and the matrix ε is given in (2).

In order to solve Eq. (4), we make some additional assumptions. First, we treat the amplitude of the wave crossing perpendicularly the film as constant and polarized in $0z$ direction. If we denote its frequency by Ω_2, then

$$\boldsymbol{E}_2 = (0, 0, E_2). \tag{7}$$

Secondly, the wave propagating along the layer is supposed to keep its initial polarization so that all the time it has two components only

$$\boldsymbol{E}_1 = (E_1^x, E_1^y, 0). \tag{8}$$

It is commonly accepted that $|\boldsymbol{E}_1| \ll |\boldsymbol{E}_2|$.

Now, we introduce circular quantities E_{R} and E_{L} (R — right, L — left), according to the formulas:

$$E_{\mathrm{R}} = \frac{1}{2}(E^x + iE^y), \quad E_{\mathrm{L}} = \frac{1}{2}(E^x - iE^y). \tag{9}$$

They fulfil the equation

$$\left(\frac{\partial^2}{\partial z^2} + \Omega_{\mathrm{R}}\right) E_{\mathrm{R}} = -2\mu_0 \chi \Omega_1^2 (|E_{\mathrm{R}}|^2 + 3|E_{\mathrm{L}}|^2) E_{\mathrm{R}}, \tag{10}$$

and analogous equation for E_L. In these equations

$$\Omega_{R,L} = \frac{1}{c^2}\Omega_1^2(1+\eta\pm\alpha B)+2\mu_0\chi\Omega_1^2 I_{tr}. \tag{11}$$

The upper sign relates to R, the lower — to L. I_{tr} denotes the intensity of the wave coming from the laser

$$I_{tr} = |E_2^z|^2. \tag{12}$$

Both circular waves may be further decomposed into two others: one propagating forwards (index F) and one — backwards (index B). So we put:

$$E_R = E_{RF}e^{i(\varphi_F - k_R z)} + E_{RB}e^{i(\varphi_B + k_R z)}, \tag{13a}$$
$$E_L = E_{LF}e^{i(\psi_F - k_L z)} + E_{LB}e^{i(\psi_B + k_L z)}. \tag{13b}$$

All the amplitudes and phases are here real numbers. The phases are meant as samll contributions to wave vectors k, corresponding to the linear case. They are given by

$$k_{R,L} = \frac{1}{c}\Omega_1(1+\eta\pm\alpha B)^{1/2}. \tag{14}$$

Hence,

$$\Omega_{R,L} = k_{R,L}^2+2\mu_0\chi\Omega_1^2 I_{tr}. \tag{15}$$

2. Analytical considerations

Equation (10) may be partially solved by applying the slow-varying amplitude approximation. It appears (for brevity we omit here the details) that then all the amplitudes are constant and equal to their values at $z = 0$ (index 0 in the following text). On the other hand, the phases depend on z and on amplitudes and may be written as follows:

$$\varphi_F = \varphi_F^{(0)} - \delta\varphi, \quad \varphi_B = \varphi_B^{(0)} + \delta\varphi \tag{16}$$

where

$$\delta\varphi = \frac{1}{k_R}\mu_0\chi\Omega_1^2\{E_{RB}^{(0)2}+2E_{RF}^{(0)2}+3E_{LB}^{(0)2}+3E_{LF}^{(0)2}+I_{tr}\}. \tag{17}$$

Similar equations holds for $\psi_{F,B}$ (one needs merely to change $R \Leftrightarrow L$).

It is now necessary to include the boundary conditions. They may be described as follows. The wave $z = 0$ is the sum of two waves: one is coming from external source (index "in") and the other is reflected from the right boundary ($z = l$). If the reflection coefficient $\mathscr{R}$ at both ends of the nonlinear region is the same, then for $z = 0^+$, we have

$$E_{in}+\sqrt{\mathscr{R}}(E_{RB}e^{i\varphi_B^{(0)}}+E_{LB}e^{i\psi_B^{(0)}}) = E_{RF}e^{i\varphi_F^{(0)}}+E_{LF}e^{i\psi_F^{(0)}} \tag{18}$$

for the $0x$ direction, and

$$\sqrt{\mathscr{R}}(E_{RB}e^{i\varphi_B^{(0)}}-E_{LB}e^{i\psi_B^{(0)}} = E_{RF}e^{i\varphi_F^{(0)}}-E_{LF}e^{i\psi_F^{(0)}} \tag{19}$$

for the $0y$ direction. We have assumed here that the incident beam is polarized along the $0x$ axis only.

For $z = 1$ one has

$$\sqrt{\mathscr{R}}(E_{RF}e^{i(\varphi_F(l)-k_Rl)} \pm E_{LF}e^{i(\psi_F(l)-k_Ll)} = E_{RB}e^{i(\varphi_B(l)+k_Rl)} \pm E_{LB}e^{i(\psi_B(l)+k_Ll)} \tag{20}$$

(the upper sign for $0x$, lower — for $0y$ direction).

By introducing two new quantities:

$$\begin{aligned} \Delta\varphi &= \varphi_F(l)-\varphi_B(l)-\varphi_F(0)+\varphi_B(0), \\ \Delta\psi &= \text{as above, with } \varphi \Rightarrow \psi, \end{aligned} \tag{21}$$

one obtains:

$$E_{RF}^2 = \frac{1}{\mathscr{R}}E_{RB}^2 = \frac{1}{4}[1+\mathscr{R}^2-2\mathscr{R}\cos(\Delta\varphi-2k_Rl)]^{-1}I_{in}, \tag{22a}$$

$$E_{LF}^2 = \text{as above, with } R \Rightarrow L \text{ and } \varphi \Rightarrow \psi. \tag{22b}$$

I_{in} denotes here the intensity of the incident beam.

These equations together with Eqs. (16) and (17) form a closed set of algebraic equations for $\Delta\varphi$ and $\Delta\psi$.

The intensity I_{out} of the wave leaving the film $z = 1$ may be expressed by E_{RF} and E_{LF} as follows:

$$I_{out} = \mathscr{T}(E_{RF}^2+E_{LF}^2) = I_R+I_L, \quad (\mathscr{T} = 1-\mathscr{R}). \tag{23}$$

Thus,

$$\Delta\varphi = -(\mu_0\chi\Omega_1^2/\mathscr{T}k_R)(3(1+\mathscr{R})(2I_{out}-I_R)+2\mathscr{T}I_{tr}), \tag{24a}$$

$$\Delta\psi = -(\mu_0\chi\Omega_1^2/\mathscr{T}k_L)(3(1+\mathscr{R})(2I_{out}+I_R)+2\mathscr{T}I_{tr}). \tag{24b}$$

The algorithm presented above enables us to calculate numerically all the characteristics of the outgoing wave, *e.g.* its intensity and the polarization. The first one is already well examined. On the other hand, the polarization is important in many applications and our attention is limited to this aspect only.

A standard quantity describing the elliptic polarization is the coherent matrix K defined as

$$K = \begin{bmatrix} \overline{|E_x|^2}, & \overline{E_xE_y^*} \\ \overline{E_yE_x^*}, & \overline{|E_y|^2} \end{bmatrix} \tag{25}$$

where — denotes an average over time. Another way of expressing K is

$$K = \frac{1}{2}(Tr\,K)\begin{bmatrix} 1+\xi, & \xi_1+i\xi_2 \\ \xi_2-i\xi_1, & 1-i\xi_1 \end{bmatrix}. \tag{26}$$

where Stokes parameters ξ_j are connected with the goemetry of the polarization ellipse in the following way:

$$\sin 2\gamma = \xi_2, \tag{27}$$
$$\tan 2\delta = \xi_2/\xi_1, \tag{28}$$
$$p = \xi_1 + \xi_2 + \xi_3. \tag{29}$$

Here, δ denotes the angle between the longer axis of the ellipse and the $0x$ axis, $\tan\gamma = \pm b/a$ (a, b — half-axes of the ellipse, two signs correspond to two directions of circulation on the ellipse). These three parameters are easily expressed through the quantities described earlier. For brevity we omit detailed calculations (see work [3] on similar problems).

3. Results of numerical calculations

Our calculations were performed for two sets of material parameters. The first one corresponds to CS_2, the second is rather a hypothetical one. In the first case, we have put $\lambda = 1.06$ μm, $\chi = 1.299 \cdot 10^{-25}$ and the girotropic constant $\alpha = 2.16 \cdot 10^{-6}$ T^{-1}. In the second case, $\lambda = 0.5$ μm, $\chi = 10^{-23}$, $\alpha = 10^{-4}$ T^{-1}. The incoming wave was constant and its intensity was put as $I_{in} = 10^5$ W/m^2.

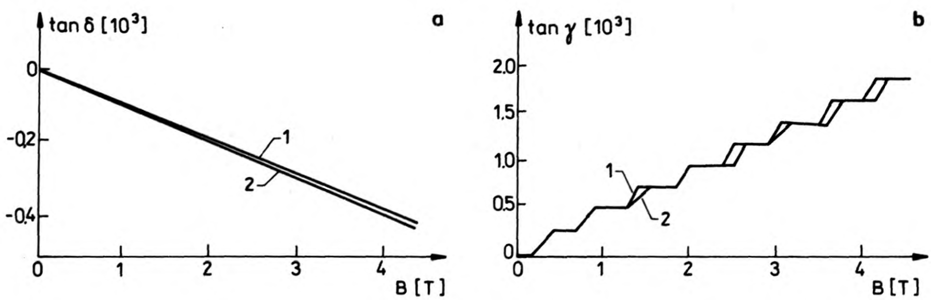

Fig. 2. Dependence of $\tan\delta$ (a) and $\tan\gamma$ (b) on B, for different values of I_{tr}. $\mathcal{R} = 0.5$, $l = 1$ mm. Physical parameters correspond to CS_2. 1 — $I_{tr} = 0.1$, 2 — $I_{tr} = 1.0$

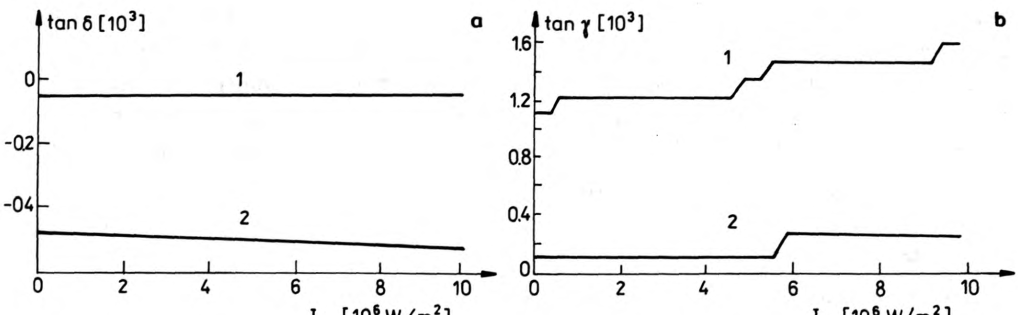

Fig. 3. Dependence of $\tan\delta$ (a) and $\tan\gamma$ (b) on I_{tr} for different values of B and for $\mathcal{R} = 0.9$ and $l = 10$ mm (CS_2). 1 — $B = 0.1$ T, 2 — $B = 1.0$ T

The most interesting results are collected in the figures. We present therein the dependence of $\tan\delta$ and $\tan\gamma$ on the intensity I_{tr} of the laser beam for different values of B and $\mathcal{R}$ and also l. Figures 2 and 3 relate to CS_2, Figures 4 and 5 are more abstract.

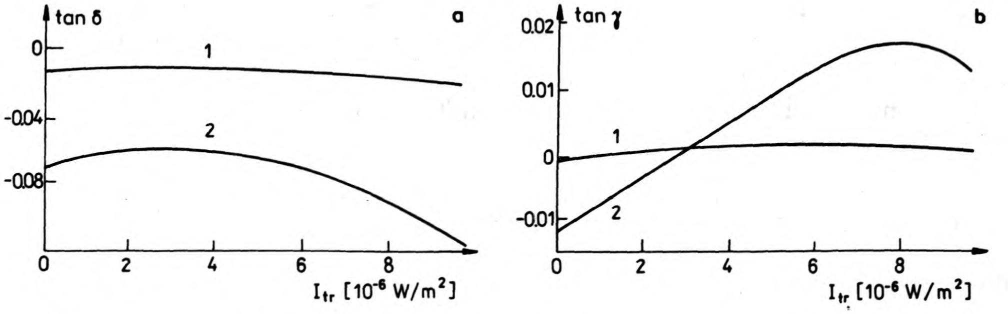

Fig. 4. Dependence of $\tan\delta$ (a) and $\tan\gamma$ (b) on I_{tr} for hypothetical material and for $\mathcal{R} = 0.2$ and $l = 1$ mm. 1 – $B = 0.1$ T, 2 – $B = 1.0$ T

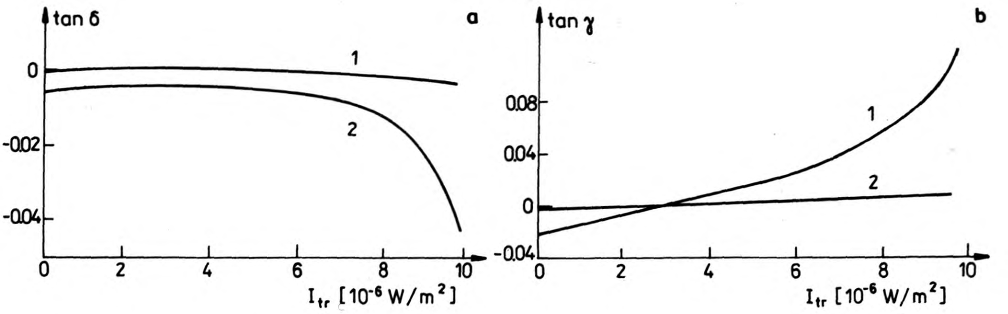

Fig. 5. As in Fig. 4, but with $\mathcal{R} = 0.9$

It is easy to show that the degree of polarization is – in our system – constant and equal to 1. This means that the outgoing beam is fully polarized.

The changes of δ and γ are, for CS_2, relatively small. For B of the order 1 T, the rotation of polarization is practically negligible. More interesting phenomena appear in media with higher values of χ and α. Increase of $\mathcal{R}$ leads to an increase of the degree of ellipticity of the wave. The transparent beam may change the direction of the rotation of E. The last effect depends periodically on I_{tr}. It is also worth noticing that parameters $\mathcal{R}$, l, and λ have essential influence on processes considered but the most important are their simultaneous changes.

References

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